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**Adaptive and Nonlinear Control Methodologies**

**S. S. Sastry**

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College of Engineering  
University of California, Berkeley, CA 94720

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*Basic Research in the Transient Behavior of Adaptive Systems.* While a number of adaptive schemes are provably convergent, their transient behavior was not well understood. Using the techniques of averaging for systems with slow parameter update we studied the rates of parameter convergence (#5), the onset of instability in an improperly excited adaptive system (#1, #14) and the frequency domain design of inputs for optimal parameter convergence (#6).

*Stabilization of Nonlinear Systems.* This is a basic unsolved problem in nonlinear control, especially when the linearization of the system is not stabilizable. In #8 and #15 we discussed the use of center manifold techniques and symptotics to devise nonlinear control laws to stabilize these systems.

*Adaptive Control of Nonlinear Systems, Linearizable by State Feedback.* A very large class of nonlinear systems is explicitly and exactly linearizable by state feedback. The problem with this design procedure recently developed by Isidori and co-workers is that it relies on the exact cancellation of nonlinear terms. Thus, if the nonlinear terms are partially unknown, as they frequently are, this methodology cannot be used. When the nonlinearities are *parametrically* uncertain, we have used parameter adaptive control to make the cancellation asymptotically exact. Our work was motivated by an application, the adaptive control of robot manipulators. In #13, we gave the first provably convergent adaptive control law for robot manipulators and implemented it on a commercial robot, the Adept I of Adept Corporation, Sunnyvale. In later work (#2), we have extended this to give a general methodology of the adaptive control of this class of nonlinear systems. In the extension of the work reported on this grant being carried out under the follow-up ARO grant DAAL03-88-K-0106. We are applying and extending this methodology for the flight control of a VTOI. aircraft (the Harrier) and possibly helicopter borne gun mounts.



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## ARO Final Report on Grant #DAAG29-85-K-0072

### Adaptive and Nonlinear Control Methodologies

We conducted a comprehensive program of research in adaptive and nonlinear control and as a result, produced three Ph.D. dissertations and a monograph either wholly or partially supported by this grant. They are respectively:

- 1) E.W. Bai, "Perspectives in Adaptive Systems: Frequency Domain Analysis and Use of Prior Information", July 1987, Ph.D. dissertation.
- 2) L.C. Fu, "Frequency Domain Techniques for the Analysis and Synthesis of Adaptive Systems," August 1987, Ph.D. dissertation.
- 3) J.E. Mason, "Identification of Continuous Time Systems in the Presence of Unmodeled Dynamics," April 1988, Ph.D. dissertation.
- 4) S. Sastry and M. Bodson, "Adaptive Control: Stability, Parameter Convergence and Robustness," Prentice Hall, to appear November 1988 (Research Monograph).

In addition there were fourteen research papers which were published in the course of this grant. They are as follows:

### Publications

1. J.E. Mason, E.W. Bai, L.C. Fu, M. Bodson and S.S. Sastry, "Analysis of Adaptive Identifiers in the Presence of Unmodelled Dynamics," Proc. of the IEEE Conference on Decision and Control, Los Angeles, Dec. 1987, to appear IEEE Trans AC, Vol. 33, No. 9, September 1988.
2. S. Sastry and A. Isidori, "Adaptive Control of Linearizable Systems," to appear in the Proc. of the Nantes Workshop on Nonlinear Control, June 1988, submitted to IEEE Trans AC., UCB/ERL Memo M87/53, 9 June 1987.
3. E.W. Bai and S.S. Sastry "Global Stability Proofs for Continuous-Time Indirect Adaptive Control Schemes," IEEE Trans AC, Vol. 32, pp. 537-543, June 1987, UCB/ERL Memo No. M86/20, March 1986.
4. E.W. Bai, "Switching Control for a Repetitive Trajectory under Parameter Uncertainty," submitted to IEEE Trans AC, May 1987.
5. E.W. Bai, L.C. Fu and S.S. Sastry, "Averaging Analysis for Discrete Time and Sampled Data Adaptive Systems," IEEE Trans on Circuits and Systems, Vol. 35, pp 137-149, February 1988, UCB/ERL Memo No. M86/67, 2 September 1986.

6. L.C. Fu and S. S. Sastry, "Frequency Domain Synthesis of Optimal Inputs for Adaptive Identification and Control," Proceedings of the American Control Conference, Minneapolis, June 1987, submitted to IEEE Trans AC., UCB/ERL Memo M87/3, 6 February 1987.
7. E.W. Bai and S.S. Sastry, "Adaptive Stabilization of Sampled Systems," Systems and Control Letters, Vol 8, No. 4, pp. 389-395, 1987, UCB/ERL Memo No. M86/52, 11 June 1986.
8. S. Behtash and S. Sastry, "Stabilization of Nonlinear Systems with Uncontrollable Linearization," IEEE Trans AC, Vol. 33, No. 6, pp. 585-590, June 88, UCB/ERL Memo No. M86/71, 30 June 1986.
9. E.W. Bai and S.S. Sastry, "Parameter Identification using Prior Information," Int Journal Control, Vol. 44, No. 2, pp. 455-473, 1986, UCB/ERL Memo No. M85/81, October 1985.
10. E.W. Bai and S.S. Sastry "Discrete Time Adaptive Control Utilizing Prior Information," IEEE Trans on AC, Vol. AC-31, pp. 779-782, August 1986, UCB/ERL Memo No. M8694.
11. E.W. Bai and S.S. Sastry "Adaptive Control of Partially Known Systems," Proc of American Control Conference, Seattle, Washington, pp. 103-108, June 1986.
12. E.W. Bai and S.S. Sastry "Adaptive Stabilization of Sampled Data Systems," Systems and Control Letters, Vol. 8, pp. 389-395, 1987, UCB/ERL Memo No. M86/52, June 1986.
13. J.J. Craig, P. Hsu, S.S. Sastry "Adaptive Control of Mechanical Manipulators," International Journal of Robotic Research, Vol. 6, No. 2, pp. 16-28, Summer 1987, IEEE International Conference on Robotics and Automation, 1986.
14. L.C. Fu and S.S. Sastry "Slow Drift Instability in Model Reference Adaptive System - An Averaging Analysis," Int J of Control, Vol. 45, No. 2, pp. 503-527, 1987, UCB/ERL Memo No. M85/25, 20 March 1986.

There were primarily four directions of research pursued under this grant. We describe them briefly here. A more complete description is in the attached abstracts.

a) *The Use of Prior Information in Adaptive Control.* A great drawback in previously existent adaptive control schemes was their neglect of prior information available to the designer. For instance, certain physical constants or plant interconnection patterns were totally neglected. This resulted in a large number of extra parameters and also a lack of robustness in the resultant controller. In a series of papers (# 3, 7, 9, 10, 11, 12) we obtained methods for getting around this drawback and gave adaptive control laws for partially known, continuous or discrete time linear systems.

b) *Basic Research in the Transient Behavior of Adaptive Systems.* While a number of adaptive schemes are provably convergent, their transient behavior was not well understood. Using the techniques of averaging for systems with slow parameter update we studied the rates of parameter convergence (#5), the onset of instability in an improperly excited adaptive system (#1, #14) and the frequency domain design of inputs for optimal parameter convergence (#6).

c) *Stabilization of Nonlinear Systems.* This is a basic unsolved problem in nonlinear control, especially when the linearization of the system is not stabilizable. In #8 and #15 we discussed the use of center manifold techniques and symptotics to devise nonlinear control laws to stabilize these systems.

d) *Adaptive Control of Nonlinear Systems, Linearizable by State Feedback.* A very large class of nonlinear systems is explicitly and exactly linearizable by state feedback. The problem with this design procedure recently developed by Isidori and co-workers is that it relies on the exact cancellation of nonlinear terms. Thus, if the nonlinear terms are partially unknown, as they frequently are, this methodology cannot be used. When the nonlinearities are *parametrically* uncertain, we have used parameter adaptive control to make the cancellation asymptotically exact. Our work was motivated by an application, the adaptive control of robot manipulators. In #13, we gave the first provably convergent adaptive control law for robot manipulators and implemented it on a commercial robot, the Adept I of Adept Corporation, Sunnyvale. In later work (#2), we have extended this to give a general methodology of the adaptive control of this class of nonlinear systems. In the extension of the work reported on this grant being carried out under the follow-up ARO grant DAAL03-88-K-0106. We are applying and extending this methodology for the flight control of a VTOL aircraft (the Harrier) and possibly helicopter borne gun mounts.

#### **Scientific Personnel Supported by this Project**

Erwei Bai (1/85 - 6/87)

Saman Behtash (7/85 - 12/86)

Li-Chen Fu (1/85 - 6/87)

C.A. Desoer (1/85 - 6/85)

Jeff Mason (1/87 - 12/87)

S.S. Sastry (1/85 - 12/87)

## **Ph.D. Abstracts**



## **Perspectives in Adaptive Systems: Frequency Domain Analysis and Use of Prior Information**

**Er-Wei Bai**

### *Abstract*

This thesis addresses frequency domain techniques and the use of prior information in the analysis of adaptive identification and control schemes.

The contribution to frequency domain analysis is twofold: First, we establish a persistency of excitation condition on the regressor vector for a reduced order identifier, i.e. it is assumed that the order of the plant is not available and only the order of the nominal model is known. The theorem states, roughly speaking, that the persistency of excitation of the regressor vector depends on the order of the nominal model, i.e. the order of the identifier and is almost independent of the existence of unmodeled dynamics. It provides a foundation for the further analysis of a reduced order identifier using averaging techniques. Then we show, under some technical conditions, that the parameter estimate will converge either to a unique tuned model or to a neighbourhood of the tuned parameter. Second, we apply frequency domain analysis techniques to the global stability proof for an indirect adaptive scheme. We present a very general indirect adaptive control scheme along with its convergence proof. We show that if the exogenous input is rich enough, then the identifier and the controller converge to their "true" values. In the thesis, only two applications have been discussed. However the scheme presented is applicable to several kinds of controller design methodologies i.e. offers a great deal of flexibility in controller design and allows for a very general richness condition on the exogenous input.

We show how prior information may be used in the analysis and application of adaptive systems by constructing a model for a wide class of partially known systems and by presenting algorithms for the adaptive identification and control of such systems. If the system is completely unknown, the methods are identical to the standard ones in the literature. However, use of the particular prior information embedded in the model results in the identification and control of a fewer number of unknown parameters and consequently better performance.

## **Frequency Domain Analysis and Synthesis Techniques for Adaptive Systems**

Li-Chen Fu

### **Abstract**

In this thesis, we present a frequency domain technique, ---averaging---, for the analysis and synthesis of both adaptive identification and control systems. A thorough analysis of adaptive systems using this technique is performed under ideal conditions as well as non-ideal conditions, with unmodelled dynamics and/or output disturbances present. With the same technique, the problem of selecting optimum inputs for the purpose of maximizing the rate of exponential convergence of adaptive parameters to their true values is solved. The outline of the dissertation is as follows.

First, a full set of averaging results for adaptive systems is provided. These results lead to stability theorems concerning exponential, partial exponential, and ultimate boundedness. Instability theorems for one- and two-time-scale dynamical systems are also given. Under the assumptions of a stationary reference input and slow adaptation, these results can be readily applied to adaptive systems.

Next, averaging is used as an approximation method to obtain estimates on the rate of parameter convergence for various adaptation algorithms. Initially, this is done for the ideal situation, and then the effects of unmodelled dynamics and output disturbances are analyzed. In particular, the notion of a "tuned model" is precisely defined. Under the assumptions of slow

adaptation and persistency of excitation, the identifier parameters are shown to converge to a ball centered at the tuned parameters whose radius goes to zero as the adaptation gain goes to zero. Similar, though slightly more complicated, results are obtained for the model reference adaptive control. A different perspective on instability for adaptive systems is also given to substantiate the importance of the choice of input signals.

Finally, a frequency domain technique for the synthesis of reference inputs for adaptive systems is proposed. An expression for what we call the *average information matrix* is derived and its properties are studied. The objective of the technique is to solve an optimization problem which maximizes the smallest eigenvalue of the average information matrix over power constrained signals. A convergent numerical algorithm is provided to obtain the globally optimal solutions. In the presence of unmodelled dynamics, a practical consideration of the frequency range in which the spectral support of the reference input should lie is also provided.

## Identification of Continuous Time Systems in the Presence of Unmodeled Dynamics

J.E. Mason

The purpose of any identification procedure is to generate a model of an unknown system. This model will then be used to study and analyze the input/output behavior of the system and/or be used in a control design procedure. In either case it is important to know in what sense the identified model represents the unknown plant and a bound on the "nearness" of the identified model to the original system.

The existence of unmodeled dynamics is inevitable in any real system. Such perturbations exist for a variety of reasons. For example, it may be that there is incomplete physical knowledge about the system, or, some effects may be small and deliberately neglected by the designer. For some systems, it is by choice that "unmodeled dynamics" are created. For example, to simplify a later design process, it is sometimes beneficial to have a lower order yet representative model of the system. For whatever reason, unmodeled dynamics will have to be dealt with in any identification/modeling process.

In the paper, Mason, Bai, Fu, Bodson and Sastry 1987 (see 1 above), we investigated the behavior of the adaptive identifier of Narendra and Kreisselmeier in the presence of unmodeled dynamics. Results from that analysis include a necessary and sufficient input richness condition to ensure that the regressor is almost always PE. This condition basically says that almost any input whose spectral measure is supported by at least  $2n+1$  points will produce a PE regressor regardless of the order of the unmodeled dynamics. Then, since there can never be fixed parameter values for the identifier which generate model matching between the identifier and the plant, one used the method of averaging to define the tuned parameters of the identifier as the equilibrium point of the averaged identifier dynamic equation. Further results were presented which show that the tuned parameters are unique when the regressor is PE and that the identifier parameters converge to a ball centered at the tuned parameters and whose radius is a strictly decreasing function of the adaptation gain. The interpretation of the tuned parameters defined for the gradient algorithm is that they are the fixed values of the identifier parameters which minimize the mean squared output error between the identifier and the unknown plant. Furthermore, it was pointed out that the tuned parameters from the gradient update are equal to the asymptotic parameter values from the standard least squares update. This paper

was concluded with an example designed to illustrate the above described results.

When an identifier attempts to identify an unknown model which has a larger degree than the identifier itself, it is in some sense trying to determine a reduced order model of that system. In our previous work we were able to determine the behavior of the identifier parameters from a strictly parameter estimation viewpoint. In the sequel to this research we decided to study the area of model reduction and apply what is learned there to the identification problem in an effort to make statements about the quality of the identification process from a model estimation standpoint. The methods of model reduction studied were

- (i) the balanced realization truncation procedure
- (ii) the Hankel norm approximation procedure
- (iii) a truncation procedure based on the balanced gains of the system and
- (iv) the  $L^2$  optimal approximation process

The first two methods above generate reduced order models which, though not guaranteed to be optimal, satisfy an  $L^\infty$  norm error bound. The third method is also not guaranteed to be optimal but does yield a reduced order model which satisfies an  $L^2$  norm error bound. The fourth method listed is an optimal approximation process but is complicated by the fact that the solution is given in the form of implicit equations.

These methods of model reduction were presented for 2 reasons. The first was to illustrate that a measure of a model's "nearness" to the original system must be clearly defined. In the area of model reduction, the two most common measures of model "nearness" and the ones used by the model reduction techniques listed above are the  $L^2$  norm of the error transfer function and the  $L^\infty$  norm of the error transfer function. It was further shown, by example, that models which are either a good  $L^2$  or  $L^\infty$  approximation may be drastically different in their appearance and in the way they represent the original system. The second reason for the presentation of the model reduction techniques was so that those techniques of analysis could be adopted for application to the identifier. The method which proved most useful in that sense was method (iv). Through the use of analytical techniques used in that model reduction process, we were able to show that when the input to the identifier was white noise, the identifier tuned parameters defined in our previous work are the parameter values which minimized the  $L^2$  norm of the error transfer function between the original system and the identifier. This however, was found to be different then finding the parameter values which produced the best  $L^2$  norm model of the original system. However, since identifier parameter behavior is dependent on the choice of identifier filter pole locations and the bandwidth of input, we were also able to show that filter pole locations have a definite affect on the quality of the resultant model. It is not clear exactly where the the best placement of the filter poles is, but examples seem to indicate that the filter poles should be placed in about the middle of the estimated bandwidth of the plant and not outside of it. Furthermore, even though this was done for white noise inputs, it was shown that use of an input with a dense enough packing of sinusoids in the bandwidth of the overall system (plant plus identifier poles) would approximate white noise results. Furthermore, it was shown that wide band inputs with proper filter pole locations are needed to generate good 2-norm models and such a combination could not be replaced by the combination of high bandwidth filter poles and a band limited input thereby illustrating the inherent difference between input spectral content and filterpole bandwidth.

## **Publication Abstracts**

## **Analysis of Adaptive Identifiers in the Presence of Unmodelled Dynamics**

*J.E. Mason, E.W. Bai, L.C. Fu, M. Bodson, and S. Sastry*

### **ABSTRACT**

In this research, we analyze the behavior of a standard identifier when the plant contains additional dynamics, called unmodelled dynamics, which invalidate the known order assumption. The first result of our analysis is an input richness condition which does not depend on the order of the unmodelled dynamics to guarantee persistency of excitation of the regressor. Then we show that the PE condition leads to a BIBO stability property for the identifier. We use the method of averaging to formally define the notion of tuned parameters as the equilibrium of the identifier averaged system. It is shown that the tuned parameters always exist and that the actual parameters converge to some neighborhood of the tuned parameters. From the definition of the tuned parameters, we derive an explicit expression to calculate them and interpret them as the fixed parameter values which minimize the mean squared output error.

## **Adaptive Control of Linearizable Systems**

**S. S. Sastry**

**A. Isidori**

### **Abstract**

In this paper we give some initial results on the adaptive control of "minimum-phase" nonlinear systems which are exactly input-output linearizable by state feedback. Parameter adaptation is used as a technique to robustify the exact cancellation of nonlinear terms which is called for in the linearization technique. We review the applications of the techniques to the adaptive control of robot manipulators. Only the continuous time case is discussed in this paper—extensions to the discrete time and sampled data case are not obvious.



## **Global Stability Proofs for Continuous Time Indirect Adaptive Control Schemes**

*Er-Wei Bai and Shankar S. Sastry*

### **ABSTRACT**

The paper presents a general stability proof for continuous time adaptive control, with very general assumptions on the identifier and controller. We show that if the exogenous input signal is rich enough, then the identifier converges to its 'true' value. Boundedness of all signals and the convergence of the controller in turn follow from that of the identifier. In the paper, only two applications (pole placement and factorization approach) have been discussed, however the results are applicable to several kinds of controller design methodologies.

## Switching Control for a Repetitive Trajectory under Parameter Uncertainty

*Er-Wei Bai*

### ABSTRACT

The most popular design methodologies dealing with parameter uncertainties are robust control, adaptive control and learning control. A robust controller is a fixed (parameter) controller which takes care of all members of a given set which parameters lie in. However the unknown parameter has to be close enough (in some sense) to some fixed nominal value. An adaptive controller is a special nonlinear controller which deals with all members of a wide class of uncertainty. But, it involves extensive real-time computation since it is basically an automatically tuning controller based on system identification. So far in much of the existing literature on adaptive control, plants considered have to be linear. The idea of a learning controller is that the input to the plant is to be corrected based on the previous operation in order to make the plant output approach the desired one. While it may be promising, a synthesis methodology is restricted to linear plants, only very recently effort has been made towards to 'nonlinear learning control'. A new scheme is presented in this note to deal with parameter uncertainty which is quite practical and rigorous. We show in this note, under some technical conditions, that after finite search steps an input  $u(t)$  will be obtained so that the plant output follows a given reference trajectory within a prescribed error bound.

## Averaging Analysis for Discrete Time and Sampled Data Adaptive Systems

*Er-Wei Bai, Li-Chen Fu and S. Shankar Sastry*

### ABSTRACT

We extend our earlier continuous time averaging theorems to the non-linear discrete time case. We use theorems for the study of the convergence analysis of discrete time adaptive identification and control systems. We also derive instability theorems and use them for the study of robust stability and instability of adaptive control schemes applied to sampled data systems. As a by product we also study the effects of sampling on unmodeled dynamics in continuous time systems.

## Frequency Domain Synthesis for Adaptive Systems

*Li-Chen Fu and S. Sastry*

### ABSTRACT

In this work, we precisely formulate the input design problem of choosing proper inputs for use in SISO Adaptive Identification and Model Reference Adaptive Control algorithms. Characterization of the optimal inputs is given in the frequency domain and is arrived at through the use of *averaging theory*. An expression for what we call the *average information matrix* is derived and its properties are studied. To solve the input design problem, we recast the design problem in the form of an optimization problem which maximizes the smallest eigenvalue of the average information matrix over power constrained signals. A convergent numerical algorithm is provided to obtain the global optimal solution. In the case where the plant has unmodelled dynamics, a careful study of the *robustness* of both Adaptive Identification and Model Reference Adaptive Control algorithms is performed using *averaging theory*. With these results, we derive a bound on the frequency search range required in the design algorithm in terms of the desired performance.

## **Adaptive Stabilization of Sampled Systems**

*Er-Wei Bai and S.Shankar Sastry*

### **ABSTRACT**

A simple discrete adaptive control scheme is proposed in this paper for stabilizing minimum phase continuous time systems under fast sampling. Even though the sampled system is not necessarily minimum phase, information about the pole-zero locations of the sampled system can be incorporated to complete the proof of stability.

## STABILIZATION OF NONLINEAR SYSTEMS WITH DEGENERATE LINEARIZATION

*S. Behlash\* and S. Sastry\**

Department of Electrical Engineering and Electronics Research Laboratory  
University of California, Berkeley, CA 94720

### ABSTRACT

We consider the problem of local stabilization of nonlinear control systems whose linearizations contain uncontrollable modes on the  $j\omega$ -axis. A general methodology for designing a stabilizing control is presented. It involves the following steps: 1) Reduction of the stability problem to the stability of the center manifold system. 2) Simplification of the vector field on the center manifold using the theory of normal forms. 3) Finding conditions under which the simplified vector field is asymptotically stable. Following these steps, three cases of degeneracies in the linearized system are treated and necessary and sufficient conditions for the existence of stabilizing controls are given in each case. Finally a theorem is presented regarding the robustness of the above control strategies.

## Parameter Identification Using Prior Information

*E.W. Bai and S.S.Sastry*

We consider the problem of identifying some unknown gain parameters in a SISO transfer function which is written as the ratio of proper stable rational functions. The technique is a generalization of currently available techniques. However, usage of the particular structure embodied in the model results in the identification of a fewer number of unknown parameters. Thus it has faster convergence rate and is less susceptible to error caused by the unmodeled dynamics. We give the proof of the convergence and show the robustness of our scheme. Finally extensions to the discrete time system and some multivariable systems are also covered.

## Discrete Time Adaptive Control Utilizing Prior Information

*E.W. Bai and S.S.Sastry*

We present schemes for discrete time adaptive control of a linear time invariant partially known system. The novelty of the paper is the set up which enable us to highlight the simplicity, to indicate the reduced dimension, to point out the identifiability condition and to specify the adaptive algorithm and convergence conditions.

## **Adaptive Identification and Control of Partially Known Systems ( Bai and Sastry)**

In recent years, several adaptive schemes have been analyzed under somewhat idealized assumptions. However, in practice, one needs to be aware of the following three avenues of research.

- 1) Robustness. We need algorithms capable of dealing with some degree of uncertainty in model and small disturbance.
- 2) Low order controllers(identifiers) to control(identify) high order plants.
- 3) Transient performance. Current schemes guarantee that the error term ( output error, parameter error etc.) go to zero as time  $t$  goes to infinity. In practice, it is always highly desirable to have an algorithm which produces small transients.

In most cases, prior information is available about the physical plant. Consequently it is plausible that the controllers(identifiers) could be low order and robust, if this prior information can be incorporated into the adaptive algorithm.

Motivated by this observation, in [1], we consider the problem of identifying some unknown gain parameters in a SISO transfer function which is written as the ratio of proper stable rational functions. The technique is a generalization of currently available techniques. However, usage of the particular structure embodied in the model results in the identification of a fewer number of unknown parameters. Thus it has faster convergence rate and is less susceptible to error caused by the unmodeled dynamics. We give the proof of the convergence and show the robustness of our scheme. Finally extensions to the discrete time system and some multivariable systems are also covered.

In paper [2], we present schemes for discrete time adaptive control of a linear time invariant partially known system along with an analysis of their convergence properties. The novelty of the paper is the set up which enable us to highlight the simplicity, to indicate the reduced dimension, to point out the identifiability condition and to specify the adaptive algorithm and convergence conditions. An extension of the results to continuous time systems has been made in [3].

## **Adaptive Stabilization of Sampled Systems** (Bai and Sastry)

In [5], we proposed an indirect adaptive scheme for the control of minimum phase continuous time systems under fast sampling. The subject is directly motivated by a fact that all continuous time systems with relative degree large than 1 will give rise to sampled systems with unstable zeros when the sampling period is sufficient small. The principal difficulty associated with indirect approach arises from the fact that the estimated system may have unstable pole-zero cancellations. Thus in the literature, more or less stringent conditions on the plant have to be imposed. Fortunately, a great deal of prior information is available about the pole-zero locations. We incorporate this prior information into the design of the controller such that the closed loop system consisting of the sampled system together with controller is exponentially stable with all signals bounded.

## **Adaptive Control of Mechanical Manipulators**

*J.J. Craig, P. Hsu and S.S. Sastry*

A major problem of designing a high performance controller for robot manipulator is the imprecise knowledge of manipulator parameters, such as mass of payload, friction coefficient. With this problem, designer is forced to sacrifice the control loop gain for stability. Which in turn leads to low disturbance rejection and poor tracking performance. We present an adaptive version of the computed torque method for the control of manipulators with rigid links. The algorithm estimates parameters on-line which appear in the non-linear dynamic model of the manipulator, such as load and link mass parameters and friction parameters and uses latest estimates in the computed torque servo. We present what we believe is the first globally convergent, rigorous proof of the stability of such a scheme in its non-linear setting, as well as its asymptotic properties and conditions for parameter convergence. We illustrate the theory with some simulation results.

## **Slow-Drift Instability in Model Reference Adaptive Systems** **An Averaging Analysis**

*Li-Chen Fu and Shankar Sastry*

The paper presents instability theorems for one- and two-time scale time-varying non-linear systems using averaging theory. These theorems are then applied to the model reference adaptive control system with unmodelled dynamics and output disturbance to analyse the mechanism of slow-drift instability.